Brans-Dicke Theory in Anisotropic Model with Viscous Fluid

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In this paper we have considered an anisotropic space-time model of the Universe in presence of Brans-Dicke (BD) scalar field ϕ, causal viscous fluid and barotropic fluid. We have shown that irrespective of fluid the causality theory provides late time acceleration of the Universe. If the deceleration occurs in radial direction and acceleration occurs in transverse direction then the anisotropic Universe will accelerate for a particular condition of the power law representation of the scale factors.

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I. INTRODUCTION

The standard cosmological models (SCM) only describe decelerated Universe models and so cannot reproduce the results coming from the recent type Ia supernovae observations upto about $z \sim 1$ [1] which in favour of an accelerated current Universe. The recent extensive search leads to some matter field which gives rise to an accelerated expansion for the Universe. This type of matter is called Q-matter. This Q-matter behaves like a cosmological constant [2] by combining +ve energy density and negative pressure. This Q-matter is either neglected or unknown responsible for this present Universe. At the present epoch, a lot of works have been done to solve this quintessence problem. Most popular candidates for Q-matter are formatted by scalar field having a potential, generates a sufficient negative pressure. Furthermore, observations reveal that this unknown form of matter properly referred to as the *dark energy* which capture almost 70% of the Universe. This is confirmed by the very recent W M A P data [3]. A large number of possible candidates for this dark energy component has already been proposed and their behaviour have been studied extensively [4]. Most of these models fit only to spatially flat ($k = 0$) Friedmann-Robertson-Walker model [5], though a few models [6] work for open Universe ($k = -1$) also.

Recently, a lot of interest have shown by the researchers in the Brans-Dicke scalar tensor theory, because of its important possible role in inflationary scenario [7]. Brans- Dicke (BD) theory is proved to be very effective regarding the recent study of cosmic acceleration [8]. BD theory is explained by a scalar function ϕ and a constant coupling constant ω, often known as the BD parameter. This can be obtained from general theory of relativity (GR) by letting $\omega \to \infty$ and $\phi = $ constant [9]. This theory has very effectively solved the problems of inflation and the early and the late time behaviour of the Universe. Banerjee and Pavon [8] have shown that the BD scalar tensor theory can potentially solve the quintessence problem. The generalized BD theory [10] is an extension of the original BD theory with a time dependent coupling function ω. In Generalized BD theory, the BD parameter ω is a function of the scalar field ϕ. Banerjee and Pavon have also shown that the generalized BD theory gives rise to a decelerating radiation model where the big-bang nucleosynthesis scenario is not adversely affected [8]. Modified BD theory with a self-interacting potential have also been introduced in this regard. Bertolami and Martins [11] have used this theory to present an accelerated Universe for spatially flat model. All these theories conclude that ω should have a low negative value in order to solve the cosmic acceleration problem. This contradicts the solar system experimental bound $\omega \geq 500$. However Bertolami and Martins [11] have obtained the solution for accelerated expansion with a potential ϕ^2 and large $|\omega|$. Although they have not considered the positive energy conditions for the matter and scalar field.

Scalar dissipative in cosmology may be treated via the relativistic theory of bulk viscosity [12, 13]. The causal and stable thermodynamics of Israel and Stewart provide a satisfactory replacement of the unstable and non-causal theories of Eckart and Landau and Lifshitz. If viscosity driven inflation occurs then this necessarily involves non-linear bulk viscous pressure [12]. A non-linear generalized Israel-Stewart theory is a more satisfactory model of viscosity driven by inflation. The inflationary Universe scenario can solve some

outstanding problems [14] of standard big-bang cosmology. Ellis and Madsen [15] have considered an FRW model with a minimally coupled scalar field along with a potential and perfect fluid in the form of radiation. For brief but comprehensive reviews, we refer to the papers by Maartens [12], Hiscock and Lindblom [16] and Lindblom [17]. In a recent communication, Zimdahl [13] discussed how the truncated theory is needed a good limit to the full causal theory in certain physical situations. Banerjee et al [18] have obtained the exact solution in BD theory in presence of causal viscous fluid.

II. BASIC EQUATIONS AND SOLUTIONS

The Brans-Dicke (BD) theory is described by the action (choosing $8\pi G = c = 1$) given by

$$S = \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} \phi^{,\alpha} \phi_{,\alpha} + \mathcal{L}_m \right] \tag{1}$$

where ϕ is the BD scalar field and ω is the BD parameter. The matter content of the Universe is composed of perfect fluid and viscous fluid given by

$$T_{\mu\nu} = (\rho + p + \pi) u_\mu u_\nu + (p + \pi)\, g_{\mu\nu} \tag{2}$$

where $u_\mu\, u^\nu = -1$ and ρ, p are respectively energy density and isotropic pressure of perfect fluid and π is the bulk viscous stress.

From the Lagrangian density (1) we obtain the field equations as

$$G_{\mu\nu} = \frac{\omega}{\phi^2} \left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha} \right] + \frac{1}{\phi} \left[\phi_{,\mu;\nu} - g_{\mu\nu} \,\Box\, \phi \right] + \frac{1}{\phi} T_{\mu\nu} \tag{3}$$

and

$$\Box\, \phi = \frac{1}{3 + 2\omega} T \tag{4}$$

where $T = T_{\mu\nu} g^{\mu\nu}$.

We now consider a homogeneous and anisotropic space-time model described by the line element

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 d\Omega_k^2 \tag{5}$$

where a and b are functions of time t alone: we note that

$$d\Omega_k^2 = \begin{cases} dy^2 + dz^2, & \text{when} \quad k = 0 \quad \text{(Bianchi I model)} \\ d\theta^2 + sin^2\theta d\phi^2, & \text{when} \quad k = +1 \quad \text{(Kantowaski-Sachs model)} \\ d\theta^2 + sinh^2\theta d\phi^2, & \text{when} \quad k = -1 \quad \text{(Bianchi III model)} \end{cases}$$

Here k is the curvature index of the corresponding 2-space, so that the above three types are described by Thorne [19] as flat, closed and open respectively.

Now, in BD theory, the Einstein's field equations for the above space-time symmetry are

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} = -\frac{1}{(3 + 2\omega)\phi} \left[(2 + \omega)\rho + 3(1 + \omega)(p + \pi) \right] - \omega\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} \tag{6}$$

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \frac{\rho}{\phi} - \frac{k}{b^2} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \frac{\dot{\phi}}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} \tag{7}$$

and the wave equation for the BD scalar field ϕ is

$$\ddot{\phi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\dot{\phi} = \frac{1}{3 + 2\omega}[\rho - 3(p + \pi)] \tag{8}$$

The energy conservation equation is

$$\dot{\rho} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)(\rho + p) = 0 \tag{9}$$

Here we consider the Universe to be filled with barotropic fluid with EOS

$$p = \gamma\rho, \quad 0 \le \gamma \le 1 \tag{10}$$

In full causal theory of non-equilibrium thermodynamics, π is given by the equation

$$\pi + \tau\dot{\pi} = -3\zeta H - \frac{\varepsilon}{2}\tau\pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right) \tag{11}$$

where, ζ the coefficient of bulk viscosity, τ is the relaxation time for the bulk viscous effects, T is the temperature. Here, H is the Hubble parameter given by $H = \frac{\dot{S}}{S} = \frac{1}{3}(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b})$, where the average scale factor can be considered as $S = (ab^2)^{1/3}$.

For $\tau = 0$, (11) reduces to the non-causal equation $\pi = -3\zeta H$. In (11), $\varepsilon = 0$ gives the truncated theory while $\varepsilon = 1$ gives full Israel-Stewart-Hiscock theory [20]. It is to be noted that in full Israel-Stewart-Hiscock theory, the derivations from equilibrium are small, so we may assume $|\pi| \ll p$.

Now from the three equations (6) - (10), we see that there are five equations and six unknown quantities, namely the scale factors a and b, the scalar field ϕ, the density ρ pressure p and the bulk viscous stress π. So any one quantity may be chosen freely to solve the system of equations. Since the field equations contain a, b and their derivatives, so without any loss of generality, we shall assume the BD scalar field ϕ is some power of the average scale factor, i.e. $\phi = A(ab^2)^\alpha$ where A and α are constants. Substituting this value in the field equations, we see that the field equations cannot be solved due to time derivative occurs in the field equations. To solve the field equations, at least in particular, let us consider the scale factors in power law forms, given by $a = a_0 t^m$, $b = b_0 t^n$ where m, n, a_0, b_0 are positive constants.

So from the equations (6) - (10), we get

$$\beta + \frac{k}{b_0}t^{2(1-n)} = 0 \tag{12}$$

where β is given by

$$\beta = (m + 2n)^2(-\frac{\omega}{2}\alpha^2 - \omega\alpha) + (m + 2n)(\omega\alpha + m - 1) + 3n^2 \tag{13}$$

From the above we see that for expanding Universe the equation (12) must be consistent for $n = 1$ or $k = 0$ (since β is constant).

For anisotropic Universe model the Hubble parameter and the deceleration parameter are given by $H = \frac{\dot{S}}{S} = \frac{m + 2n}{3t}$ and $q = -\frac{S\ddot{S}}{\dot{S}^2} = -1 + \frac{3}{m + 2n}$.

For accelerating Universe model, we have $q < 0$. To support the acceleration we find a relation between the powers of scale factors given by $m + 2n > 3$. For $n = 1$ we have $m > 1$. But for $k = 0$ we have $\beta = 0$. In this case, $m + 2n > 3$ gives the domain of n as $n > 1 + \sqrt{\frac{2 + \omega\alpha + 3\omega\alpha^2}{2}}$ or $0 < n < 1 - \sqrt{\frac{2 + \omega\alpha + 3\omega\alpha^2}{2}}$ with $0 < \alpha < 0.52$

and $\omega > -3/2$.

From equations (7) and (8), we have the expressions of ρ and π as

$$\rho = \rho_0 t^s + \rho_1 t^r \tag{14}$$

$$\pi = \pi_0 t^s + \pi_1 t^r \tag{15}$$

where, $r = (m + 2n)\alpha - 2n$, $s = (m + 2n)\alpha - 2$ and $\rho_0, \rho_1, \pi_0, \pi_1$ are constants.

Now we assume that the coefficient of bulk viscosity ζ and the relaxation time τ are simple power function of ρ. Let, $\zeta = \zeta_0 \rho^\xi$ and $\tau = \frac{\zeta}{\rho}$ where ξ and $\zeta_0 (> 0)$ are constants.

Now integrating equation (11) with the help of equations (14) and (15), we have the expression for T as

$$LogT = -\frac{2\rho_1^{1-\xi}}{\varepsilon\zeta_0(\xi r - r - 1)} t^{1+r-\xi r} \, {}_2F_1[\frac{r - \xi r + 1}{s - r}, \xi - 1, \frac{s - \xi r + 1}{s - r}, -\frac{t^{s-r}\rho_0}{\rho_1}] - Log(\rho_0 t^s + \rho_1 t^r)$$

$$+ \left\{ \frac{2}{\varepsilon} + \frac{(\pi_1\rho_0 - \pi_0\rho_1)(s + 2)}{(s - r)\pi_0\pi_1\alpha} \right\} Log(\pi_0 t^s + \pi_1 t^r) + \left(\frac{s + 2}{\alpha} \right) \left\{ 1 + \frac{(s\pi_0\rho_1 - r\pi_1\rho_0)}{(s - r)\pi_0\pi_1}) \right\} Logt \tag{16}$$

Now we consider two cases: $n = 1$ and $k = 0$.

Case I: $n = 1$: In this case, the expressions of ρ and π can be written as

$$\rho = \rho_2 t^{p_1} \quad \text{and} \quad \pi = \pi_2 t^{p_1} \tag{17}$$

where $p_1 = (m + 2)\alpha - 2$, $\rho_2 = \rho_0 + \rho_1$ and $\pi_2 = \pi_0 + \pi_1$.

So from equation (16), we get

$$T = T_0 t^A e^{Bt^{p_1 - p_1\xi + 1}} \tag{18}$$

where $A = \frac{2p_1}{\varepsilon} + \frac{2(p_1 + 2)\rho_2}{\varepsilon\pi_2\alpha} + \frac{p_1 + 2}{\alpha} - p_1$, $B = \frac{2}{\varepsilon\zeta_0\rho_2^{\xi-1}(p_1 - p_1\xi + 1)}$ and T_0 is a constant.

CaseII: $k = 0$: In this case, the expressions of ρ and π become

$$\rho = \rho_0 t^s \quad \text{and} \quad \pi = \pi_0 t^s \tag{19}$$

So from equation (16), we get

$$T = T_0 t^{A_1} e^{B_1 t^{s - \xi s + 1}} \tag{20}$$

where $A_1 = \frac{2s}{\varepsilon} + \frac{2(s + 2)\rho_0}{\varepsilon\pi_0\alpha} + \frac{s + 2}{\alpha} - s$ and $B_1 = \frac{2}{\varepsilon\zeta_0\rho_0^{\xi-1}(s - \xi s + 1)}$.

III. DISCUSSIONS

We have considered an anisotropic space-time model in presence of Brans-Dicke (BD) scalar field ϕ, causal viscous fluid and barotropic fluid. Here, we have found the exact solution for the Bianchi-I, Kantowaski-Sachs and Bianchi-III models in a full theory of non equilibrium thermodynamics. From the equations (18) and (20)

we see that the temperature T is an explicit function of time and these values clearly inconsistent for $\varepsilon = 0$. But for $\varepsilon = 0$, equation (11) reduces to the truncated theory. But $\varepsilon \neq 0$ support the full causal theory and in this case the temperature T is completely explicit function of time t for both the cases i.e. for $n = 1$ and for $k = 0$. For $n = 1$, we have shown that the acceleration is possible if $m > 1$. Also for $k = 0$ i.e, for flat Universe, acceleration is possible if n satisfies some restriction, which is described before. If $n < 1$, the acceleration in the radial direction is not possible, but in the transverse direction acceleration is possible as long as $m + 2n > 3$ and effectively the anisotropic Universe will be accelerate. Thus, in a full causal theory, the acceleration is possible for anisotropic Universe.

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